STRESS-STRAIN RESPONSE MODELLING OF GLASS FIBRE REINFORCED EPOXY COMPOSITE PIPES UNDER MULTIAXIAL LOADINGS

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ABSTRACT
This paper presents the modelling of stress strain response of glass fibre reinforced epoxy (GRE) composite pipes subjected to multiaxial loadings at room temperature (RT). This particular modelling work was developed to predict the non-linear stress strain response caused by the fatigue cyclic and static loading in the multiaxial ultimate elastic wall stress (UEWS) tests by considering the effects of matrix cracking within the laminates. The UEWS test, whilst not yet standardized, appears to offer an attractive alternative to existing procedures of qualifying GRE pipes. The ply properties initially expressed as a function of crack density was computed as a function of increasing stress and strain using shear lag approximation. The predictions are found to be in good agreement with the data from multiaxial UEWS tests on ±55° filament wound glass-reinforced epoxy pipes.

Keywords: Stress strain response, Multiaxial loadings, Composite pipes, Cyclic and Static loading, Crack density.

INTRODUCTION
The failure behaviour of filament wound GRE pipes subjected to biaxial load has been the subject of numerous experimental and modelling investigations spanning decades, as demonstrated in the literature (Hull, Legg et al. 1978; Frost and Cervenka 1994; Hale, Shaw et al. 2000; Gibson, Saied et al. 2003; Tarakcioglu, Gemi et al. 2005; Mertiny and Ellyin 2006; Carvalho and Marques 2007; Meijer and Ellyin 2008). The majority of such investigations have emphasized on the failure envelopes, fatigue strength, leakage and the associated deformation of angle ply laminates similar to those used in GRE pipes. However, whilst most of these studies concentrated on structural failure in composite pipes, the more significant issue of micro structural progressive damage, which leads to the final failure, is less clear.

Most of the literature has reported that filament wound composite pipes under fatigue biaxial load failed due to sequences of damage which involve transverse matrix cracking, delamination, weepage and fibre fracture (Jones and Hull 1979). This is illustrated in Figure 1 (Reifsnider, Henneke et al. 1983). Increases in applied load cause the continuous nucleation and accumulation of transverse matrix cracks along the fibre direction. When the matrix crack density reaches saturation, delamination, weepage and fibre fracture may occur hence causing a rapid progression of damage leading to final catastrophic failure.
Matrix cracking within composite laminates has been recognized as the major factor causing the reduction in stiffness of laminates. Various models have been presented to characterize such degradation in stiffness due to transverse matrix cracking under in-plane uniaxial and multiaxial loading. Among these models are the ply-discount approximation (Hanh and Tsai 1974), the continuum damage model (Nairn and Hu 1994; Li, Reid et al. 1998), shear lag model (Highsmith and Reifsnider 1982; Norman and Dvorak 1988), self-consistent scheme (Laws, Dvorak et al. 1983), and the variational model proposed by Hashin (Hashin 1985; Praveen and Reddy 1998). Recently, Katerelos et al. (Katerelos, McCartney et al. 2006) conducted an analysis of the effect of matrix cracking on the behaviour of angle ply laminates loaded statically using the equivalent constraint model (ECM). The approach showed a good agreement with the experimental results obtained by microscopic strain measurement using the laser Roman spectroscopy technique (Katerelos, Lundmark et al. 2007).

A finite element model was proposed by Tao and Sun (Tao and Sun 1996) and Sun and Tao (Sun and Tao 1998), who investigated the effects of matrix cracking on the stiffness degradation of laminates. The predicted normalized transverse and shear modulus was later plotted against the exponential function of the normalized crack density of a cracked lamina. The authors concluded that normalized crack density rather than crack density is a more appropriate parameter to be used in predicting cracking damage. Frost et al. (Frost and Cervenka 1994) studied the influence of loading frequency in predicting the long term fatigue behaviour of a GRE pipe. They concluded that the prime failure mechanism observed for short and long term fatigue were leakage as a result of matrix cracking. As mentioned earlier, although transverse matrix cracking may not cause abrupt structural damage in pipeline applications, it is highly detrimental since it leads to weepage failure which, if not treated, can trigger the development of other, more deleterious forms of damage such as fibre breakage or bursting.

This present investigation models the stress strain response of GRE pipes as results of transverse matrix cracking during multiaxial UEWS test. The results then were compared with the experimental UEWS data to achieve the closest fit.
ULTIMATE ELASTIC WALL STRESS (UEWS)

While the current procedure for qualifying GRE pipes based on regression analysis provides very good predictions of the long term behaviour of the pipe, manufacturers are driven by the need for a faster and simpler qualification process. A number of possible short term tests have been examined such as the interlaminar shear stress (ILSS), flexural and UEWS tests. The UEWS test is not yet standardized, but appears to offer an attractive alternative to existing procedures. Its principles were first investigated by Shell Research in 1968 (Schwencke 1968). There are very limited studies that have been reported on this procedure. Hull (Hull, Legg et al. 1978) and Frost (Frost and Cervenka 1994) have both reported on the UEWS test and observed that matrix cracking is consistently associated with non-linearity in stress strain responses.

The intention of the UEWS test is to identify, by examining the stress-strain response, a stress level below which damage growth is either negligible or at least sufficiently low to prevent long term failure at the design life. GRE pipe fails when debonding occurs between the fibres and matrix interface. Once debonding takes place, there will be less surface area for proper stress distribution. This leads to the development of stress concentrations within the GRE system which, in turn, causes further debonding. The point at which the fibre-matrix interface starts to debond is used as an indication of the borderline between permissible and non-permissible deformation. This point is called the Ultimate Elastic Wall Stress (UEWS).

The UEWS test involves the application of groups of 10 one-minute hydrostatic pressure cycles at increasing pressure levels. The strain at the end of the first and the last cycle of each ten cycle group is measured, and these values are plotted against pressure (or hoop stress). If zero or negligible damage occurs at a particular pressure level, then a linear relationship is observed between strain and hoop stress, and the strain after the tenth cycle in the group is the same as at the first cycle. As the UEWS is approached, a deviation in strain can be seen between the first and the last cycle, and the relationship begins to become non-linear. This non-linearity in the stress–strain relationship will then be used to indicate the UEWS point which corresponds to first ply failure in the pipe. Further details on the UEWS test procedure and the calculations involved are given in recent paper by Abdul Majid et al. (Abdul Majid 2011).

THE MODEL

Preceding to the UEWS testing, the theoretical mechanical properties of the individual ply and the laminates or the pipe was calculated and later compared with the experimental results. In the approach, micromechanics theory was used to derive the elastic properties of the unidirectional ply from those constituent composites using Hill and Halpin-Tsai simplification analysis formulation.

First, for the calculation of reinforcement fibre, rule of mixtures was used to predict the $E_1$ and $v_{12}$ to a good accuracy. However, the same treatment on predicting $E_2$ gives a large error due to the non-uniform distribution of stress and strain in transverse direction. Hence, Halpin-Tsai simplification was used instead to calculate the $E_2$ and $G_{12}$ of the ply. Based on isotropic glass fibre reinforcement properties provided by FPI for the Wavistrong pipe product; $E_g = 73$ GPa and $v_g = 0.59$, epoxy matrix properties; $E_m = 3.6$ GPa and $v_m = 0.41$, the ply properties were calculated to be $E_1 = 44.5$ GPa, $E_2 = 12.2$ GPa, $G_{12} = 4.33$ and $v_{12} = 0.28$. The properties of the ±55° GRE pipe were then computed using laminate theory and given in the axial and hoop direction of the pipe.
The details on laminate theory is not discussed here but is derived in details in most composite laminate resources. From the calculation,

\[ E_{\text{axial}} = 11.52 \text{ GPa} \quad E_{\text{hoop}} = 19.70 \text{ GPa} \]

\[ v_{\text{axial}} = 0.40 \quad v_{\text{hoop}} = 0.69 \]

\[ G_{12} = 11.76 \text{ GPa} \]

Also important to note that, radial component in this case is much lower than that of axial and hoop components and therefore, ignored.

**Stress-strain modeling**

In this section, the overall stress strain curves of UEWS test at various stress ratios were modelled by implementing ‘superposition’ of linear and non-linear response predicted separately. Under increasing group cycle pressure, the axial and hoop strains computed from elastic analysis were simply superposed together with the strains caused by the stiffness degradation due to the transverse matrix cracking.

For internal pressure loading only of filament wound GRE pipes, the stress is calculated from the following equation;

\[ \sigma_H = \frac{Pd}{2t} \]
\[ \sigma_A = \frac{Pd}{4t} \]

The corresponding strains produced by the these stresses generated in the tubes is then worked out from the following relations;

\[ \varepsilon_H = \frac{\sigma_H}{E_H} - v_{HA} \frac{\sigma_A}{E_A} \]
\[ \varepsilon_A = \frac{\sigma_A}{E_A} - v_{AH} \frac{\sigma_H}{E_H} \]

These strains were then transformed to the ply coordinate system by multiplying with the transformation matrix. Hence,

\[ \varepsilon_1 = \varepsilon_A \cos^2 \theta + \varepsilon_H \sin^2 \theta \]
\[ \varepsilon_2 = \varepsilon_A \sin^2 \theta + \varepsilon_H \cos^2 \theta \]
\[ \gamma_{12} = 2 \sin \theta \cos \theta (\varepsilon_H - \varepsilon_A) \]

Since the pipe wall is an angle ply laminates, the lamina can be considered of having orthotropic elastic properties, which are highly dependent on the winding angle \( \theta \). Thus, the stress-strain response at a low stress level of which the stress strain behaviour can be considered to be linear, the stresses in the unidirectional ply can be written as follows;

\[
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{pmatrix} =
\begin{pmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{pmatrix}
\]

Where \( Q_{ij} \), \( Q_{12} \) and etc. are the stiffness matrixes, which can be expressed in engineering terms as,
\[
Q_{11} = \frac{E_1}{1 - v_{12}v_{21}}; \quad Q_{12} = \frac{v_{12}E_1}{1 - v_{12}v_{21}} = \frac{v_{21}E_2}{1 - v_{12}v_{21}}
\]
\[
Q_{22} = \frac{E_2}{1 - v_{12}v_{21}}; \quad Q_{66} = G_{12}
\]

Where, \(E_1\) and \(E_2\) are the modulus of elasticity in the lamina’s principal axes. However, due to the significant differences in the thermal expansions between the reinforcement materials and the matrix, it is often causing generation of residual stress prior to actual loading on the pipe. So, it is imperative that this thermal stress generated is taken into account when modeling the stress strain response.

**Non-linear response**

Previous section described the stress strain response of a GRE pipe at low strain level, which also took into account the effect of thermal stress. However, it is well acknowledged that at a higher strain level, there are obvious non-linearity existed. In this modeling work, the non-linearity prediction takes into account of the effects of transverse matrix cracking during UEWS testing towards degradation of elastic properties of the pipes. The predictions then plotted against the experimental strain data taken from the 10th cycle of the UEWS tests of various hoop to the axial ratio.

It is well discussed that the failure of GRE pipe is primarily controlled by the transverse matrix crack within the laminates, which occur long before the final failure. The model developed here is used to predict the crack density of tubes as the function of applied stress, hence the subsequent non linearity of stress strain relationship caused by the initiation and progression of a matrix cracking under increasing pressure. From finite element model developed by Sun and Tao, the deterioration in the transverse and shear modulus of composite laminates due to the increasing presence of matrix cracks can be estimated in the form of;

\[
\frac{E_2}{E_2^0} = \exp(-\alpha_{E2}\rho^*)
\]
\[
\frac{G_2}{G_2^0} = \exp(-\alpha_{G}\rho^*)
\]

Where;

\(E_2\) and \(E_2^0\) are effective and initial transverse modulus of ply respectively.

\(G_2\) and \(G_2^0\) are effective and initial shear modulus of ply respectively.

\(\alpha_{E2}\) and \(\alpha_{G}\) are curve fitting constant.

\(\rho\) is the normalized crack density function.

In this model, the non-linearity response as a result of matrix micro cracking only took place when the transverse stress in the ply reached the failure strength of the epoxy resin. Hence, the relationship between the crack density and applied stress can be derived (Roberts, Evans et al. 2003) and given below;

\[
\rho = \kappa \left[ \frac{\sigma_2 - \sigma_2^{fail}}{\sigma_2^{fail}} \right]
\]

Where; \(\sigma_2\) is the limiting transverse stress in unidirectional ply

\(\sigma_2^{fail}\) is the failure strength of the matrix material
\[ K = \sqrt{\frac{(E_1 + E_2)G}{E_1E_2}}, \] where K involves only the ply modulus constants

The estimation of effective transverse and shear modulus of the ply at every pressure group increment can then be calculated from equation (8). For close adaptation to the experimentally determined curve of all stress ratios, the curve fitting constants \( \alpha_{E_2} \) and \( \alpha_G \) were fitted by optimising one constant at a time while retaining the value of the other. \( \sigma_{2\text{ fail}} \), which is transverse failure stress was adjusted and assigned to a constant value hence demonstrating the effects of total stress on the laminate (Roberts, Evans et al. 2003). The effective modulus then applied with the laminate theory to determine the new corresponding axial and hoop modulus of the pipe after taken into account the effects of the matrix cracking. The gradually degraded stiffness calculated was later inserted into equation (2) establishing the nonlinear stress strain response.

**RESULTS AND DISCUSSIONS**

The laminate properties listed in Table 1 based on the ply properties of angle ply laminate layup similar to the ±55° GRE pipe used in this investigation were calculated using the laminate theory. However, often, the resin rich top coating of the pipes was ignored as the structural element during calculation. The comparison between the mechanical properties obtained analytically and those by experimental means provided by FPI shows an acceptable agreement. This proves that the top coating on the outer surface of the pipe has a very minimal effect on the overall stiffness of the pipe. Even so, it is to note that these values are very much dependent on the volume fractions and its constituent’s properties. Thus, it must be determined experimentally prior to the UEWS tests and after weepage failure.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value from laminate theory</th>
<th>Experimental value by FPI</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial modulus, ( E_{ax} )</td>
<td>11.52GPa</td>
<td>11.5GPa</td>
<td>0.2%</td>
</tr>
<tr>
<td>Hoop modulus, ( E_{hp} )</td>
<td>19.70GPa</td>
<td>19.0GPa</td>
<td>3.7%</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \nu_{ah} )</td>
<td>0.40</td>
<td>0.38</td>
<td>5.3%</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \nu_{ha} )</td>
<td>0.69</td>
<td>0.65</td>
<td>6.2%</td>
</tr>
<tr>
<td>Shear modulus, ( G_{ah} )</td>
<td>11.76GPa</td>
<td>11.0GPa</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

**Stress strain response of UEWS tests**

The modelled stress strain curves for a different ratio of fitting constants \( \alpha_{E_2}/\alpha_G \) at various ratios of UEWS tests are shown in Figure 2-5. The calculations are based on equation (8-9) before subjected to the laminate theory to determine the corresponding strains in the pipe axes. Optimizations of the ratio of the fitting constant was carried out with the intention of getting the best possible match to the experimental strains of the 10\(^{th}\) cycle obtained from UEWS test with axial strains superimposed at different \( \alpha_{E_2}/\alpha_G \) ratios. Throughout the modeling work, \( \sigma_{2\text{ fail}} \) was chosen to be between 40-50MPa, since these values give the best fit for all loading conditions.

At first qualitative judgment on the results of the model implemented, suggests that they are closely conformed to the UEWS experimental data. In all loading conditions, the non-linearity modelled indicated slow change in the slope rather than
an abrupt change in response, which normally seen and described as the knee point. Figure 2 shows the model curve fitting, together with the actual findings for UEWS test conducted at 2:1 hoop to the axial stress ratio, within the room temperature environment. As we can see from the plot, the ratio of curve fitting constants between $\alpha_{E2}$ and $\alpha_G$ from equation (8) can be seen increasing from 0.8 to 1.28 to give the best fit of the stress strain response with the experimental results. At the ratio of 0.8, the stress strain behaviour showed an almost linear response. At $\alpha_{E2}/\alpha_G = 1.0$, the curve in the non-linear section showed an upward shift to a higher strain value. Further increase of $\alpha_{E2}/\alpha_G = 1.28$, at the end gives good agreement on the non-linearity response to the experimental result for the case of 2:1 loading condition. Here, it appears that by increasing the $\alpha_{E2}$ constant, which relates the effects of a matrix cracking to the deterioration in transverse modulus, the model’s curve can be matched very well to the non-linear behaviour showed by the actual findings from UEWS tests.

Similar trend was also noted for the cases of 1:1 loading and pure axial (0:1) loading conditions illustrated in Figure 3 and 4. Though, the modelled strains are slightly higher to those obtained experimentally, especially within the linear region. These slight discrepancies probably are due to the errors during testing or data scatters, which exist while analyzing the data. Considerable increased values of $\alpha_{E2}/\alpha_G$ were attained for the case of these loadings. This implied that the non-linearity of the stress strain response during axial dominated loading much has been caused by the deterioration in transverse modulus. For 1:1 loading, the best fit was obtained at $\alpha_{E2}/\alpha_G = 3.0$. Whilst pure axial loading, which presumably more prone to transverse stiffness reduction by matrix cracking showed the closest fit to experimental data at $\alpha_{E2}/\alpha_G = 6.0$, which is the highest of the previous two modelling results. This is because; in this type of loading the load is very much dominated by the epoxy matrix.

![Figure 2](image.png)

**Figure 2.** Experimental and model stress strain curve for UEWS test (2:1) at room temperature.
Figure 3. Experimental and model stress strain curve for UEWS test (1:1) at room temperature.

Figure 4. Experimental and model stress strain curve for UEWS test (0:1) at room temperature.

Figure 5. Experimental and model stress strain curve for UEWS test (1:0) at room temperature.
All in all, the failure strains for the three loading conditions were modelled close to their experimental data, at 0.2% for 2:1 loading, whereas in 1:1 and pure axial loadings the failure strains were at 0.4% and 0.6% respectively. The change in the strains from linear to a non-linear response in these loadings can be said is due to the reduction in transverse stiffness as the crack density increases leading to weepage failure.

On contrary to previous results, for pure hoop loading (1:0), the ratio between $\alpha_{E2}$ and $\alpha_G$ showed a reduction from 1.0 to 0.625 to achieve the best fit. As shown in Figure 5, at $\alpha_{E2}/\alpha_G=1.0$, a practically linear stress strain behaviour was established. Reducing the ratio of the fitting constant to 0.8 caused a downshift of the hoop strains indicating the starts of the non-linear response, closer to the experimental results. Finally, optimizations is achieved at $\alpha_{E2}/\alpha_G = 0.625$. This suggests that, unlike previous results, for hoop dominated loading the fitting constant $\alpha_G$ that relates the deterioration of shear modulus, is more sensitive in causing the non-linear response outcome of the strains. UEWS points for this loading was taken at $\sigma_H = 220\text{MPa}$, which later transformed to the ply stresses and resulted in $\tau_{12}=220\text{MPa}$. It is believed that at this stress, it is sufficient to cause shear failure in the resin system.

CONCLUSIONS

The stress-strain response as results of increasing transverse matrix cracking of GRE composite pipes under multiaxial UEWS tests is presented in this paper. The plots show that the model developed from the classical laminate theory which takes into account the effects of transverse matrix microcracks on stiffness and strains is capable of predicting the resulted elastic properties. The results from the model for all stress ratios showed a good agreement with the experimental data. The ratio of curve fitting constants between $\alpha_{E2}$ and $\alpha_G$, which relates the effects of a matrix cracking to the deterioration in transverse modulus for hydrostatic loading (2:1) and axial dominated loadings (1:1 and 0:1) were found to increase and noted to become more pronounced at axial dominated of pure axial loading (0:1). On the contrary, modeling for pure hoop loading (1:0) showed a reduction in the ratio between $\alpha_{E2}$ and $\alpha_G$ from 1.0 to 0.625 to achieve the closest agreement to experimental data. This seems to indicate that the fitting constant $\alpha_G$ that describes the degradation of shear modulus is more sensitive in causing the non-linear response outcome of the strains.

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